

ORIGINAL ARTICLE

Brain Functional Connectivity During a Mental Arithmetic Task Measured by Partial Directed Coherence

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ABSTRACT

Introduction: Researchers generally concentrate on executive function since it can both improve human capacities and identify cognitive impairments. A person's cognitive function can be assessed using various executive function tests. Processes including inhibition, updating, and shifting are related to executive function. Mental math exercises are used as one of the updating tests. Arithmetic processing in the brain involves a network of regions. However, most existing studies on brain connectivity during arithmetic tasks have been conducted using functional magnetic resonance imaging (fMRI), while studies based on electroencephalography (EEG) connectivity remain limited. This gap highlights the need for further exploration using EEG methods. This study aims to investigate the brain's functional connectivity during an arithmetic problem using EEG technology and to compare the observed connectivity patterns with findings from previous neuroimaging studies. **Materials and Methods:** The partial directed coherence (PDC) approach is employed since it can identify direct interactions between more than two channels. EEG signals were recorded from ten healthy adults (aged 18–40 years) using 19 electrodes following the 10–20 system while subjects performed continuous subtraction tasks (starting from 1000 and subtracting 13). Signals were pre-processed to remove artifacts before calculating PDC connectivity at one-second intervals and averaged across trials. **Results:** The EEG-based PDC research showed that active destination regions for brain connections during arithmetic tasks are similar to those reported using other techniques. However, the posterior region is missing in the network of regions among the subjects involved in the study, which may be related to averaging limitations or compensatory activation in frontal regions. **Conclusion:** The present study on EEG-based PDC analysis reveals that functional connectivity during mental arithmetic tasks largely mirrors patterns observed with fMRI—predominantly engaging frontal and temporal regions. Notably, the consistent absence of parietal activation may reflect limitations in the averaging approach or indicate compensatory neural strategies. These findings underscore the potential of EEG-PDC in cognitive research and highlight the need for further investigation.

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INTRODUCTION

Executive function has been generally studied among researchers as it can enhance human skills and also help determine cognitive disabilities, such as attention deficit hyperactivity disorder (ADHD) (1), brain injury (2), oppositional defiant disorder (ODD) (3), intellectual disabilities (4), autism disorder (5), and other impairments that will affect a person's life, behaviors, and skills.

There are several assessments of executive function to evaluate a person's cognitive function and to improve the cognitive abilities of patients who have executive impairment (6,7). Executive function is associated with the shifting, updating, and inhibition processes (8-10). The updating process is a person's ability to hold previous information momentarily in short-term memory by paying attention and using the information to solve problems (9, 11, 12). One of the assessments for updating function involves using mental arithmetic tasks. Performing mental arithmetic requires a working memory load. Studies on executive function indicate that this load contributes to the demands of executive function resources and is associated with academic

performance, including mathematical achievement (13). Arithmetic processing in the brain involves a network of regions. The network is dynamic, i.e., changes over time as the network undergoes developmental changes in its function, connectivity, and structure (14). Most of the existing studies on brain connectivity during arithmetic are performed using the functional magnetic resonance imaging (fMRI) technique. It is found that fronto-parietal activity is increased in children compared to adults during arithmetic tasks. A similar study can be conducted using the electroencephalogram (EEG) technique. However, previous EEG studies on arithmetic tasks have mainly focused on event-related potentials or individual frequency bands, while the investigation of functional connectivity using EEG remains limited. This represents a gap in understanding real-time directional interactions of brain regions during arithmetic processing.

Therefore, this paper explores the brain's functional connectivity using the EEG during an arithmetic task, utilizing the partial directed coherence (PDC) approach for its ability to detect direct interactions between more than two channels. Other methods used to determine the direction of information flow between two or more signals are phase slope index (PSI) and renormalized partial directed coherence (rPDC). PSI estimates the slope of phase difference over frequencies, which indicates the direction of information flow. Meanwhile, rPDC is like PDC but deals with different frequency components. PSI and rPDC methods are not covered in the paper. PDC was selected in this study because it is based on a multivariate autoregressive model, which allows the analysis of direct interactions among multiple EEG channels simultaneously rather than only pairwise comparisons. This makes PDC more suitable for capturing complex brain network dynamics during cognitive tasks such as arithmetic, where multiple regions are expected to interact at the same time. In contrast, PSI is limited to phase-based pairwise estimations, and rPDC, while addressing frequency-specific issues, does not provide additional advantages over standard PDC for the current dataset and objectives. The purpose of this study is to identify the active brain regions involved in mental arithmetic tasks using EEG-based PDC analysis and to compare the observed connectivity patterns with those reported in previous neuroimaging studies.

METHODS

Participants

Ten normal, healthy subjects aged 18–40 years old gave written informed consent to participate in the experiment. Inclusion criteria required participants to have no history of neurological or psychiatric disorders, normal or corrected-to-normal vision, and the ability to follow task instructions. Exclusion criteria included any neurological or psychiatric illness, use of medications

affecting brain activity, excessive fatigue, or failure to comply with the experimental procedures. In addition, subjects were advised to refrain from smoking cigarettes and consuming any caffeine-based products for 8 hours prior to the data collection process, to avoid alcohol consumption for at least 24 hours, and to ensure adequate sleep the night before. The study was approved by the Medical Research and Ethics Committee under the Ministry of Health Malaysia (NMRR ID-21-02440-3BM).

Task

In the mental arithmetic task, subjects were asked to perform a mental subtraction operation starting from the number 1000. They were instructed to minus the number of thirteen (13) continuously because this subtraction sequence reduces the chance of repetitive number patterns and ensures that the unit digits take longer to repeat, thereby increasing the working memory demand of the task. EEG signals were recorded for the duration of 1 minute while the subjects performed the mental arithmetic task. At the end of the session, subjects were asked the last number obtained from the mental calculation. The task was repeated five times. The following mental arithmetic task starts with the correct number obtained in the previous trial, or if the subject gave the wrong answer, the nearest correct number will be provided to the subject. As a control condition, before each arithmetic task, participants were instructed to sit quietly and remain relaxed for 1 minute while EEG signals were recorded. This control task was used as baseline activity rather than a separate control group.

EEG recording

The International 10–20 electrode placement system was used with 19 electrodes placed on the scalp for capturing EEG signals (NVX-52 from MKS). EEG signals were recorded in reference to either A1 or A2 in accordance with the International 10-20 electrode placement system. Electrode impedances were kept below 20 k Ω . EEG signals were stored on a personal computer with a sampling frequency of 500 Hz.

Data analysis

Networks of neurons consist of nodes (channels) that communicate with one another, such as 1 to 2, 2 to 3, and 3 to 4, as illustrated in Fig. 1. In order to detect this flow, PDC is used. The PDC is computed for the estimated multivariate autoregressive coefficients in Eq. 1, as defined by (15) using Eq. 2. The coefficients $a_{ij}(r)$ are the interaction effect due to $x_j(n-r)$ on $x_i(n)$ for each delay point r . While $a_{ij}(f)$ represents the difference between the Fourier transform of the coefficient series A_r in Eq. 1 and an identity matrix of the same size as A_r , while Π_{ij} is the PDC result between 0 and 1, the higher value represents stronger functional connectivity between i and j at a frequency f .

$$A_r = \begin{bmatrix} a_{11}(r) \dots a_{1M_c}(r) & \vdots & \vdots & a_{M_c 1}(r) \dots a_{M_c M_c}(r) \end{bmatrix} \quad (1)$$

$$\Pi_{ij}(f) = \frac{|a_{ij}(f)|}{\sqrt{a_j^H(f) a_j(f)}}$$

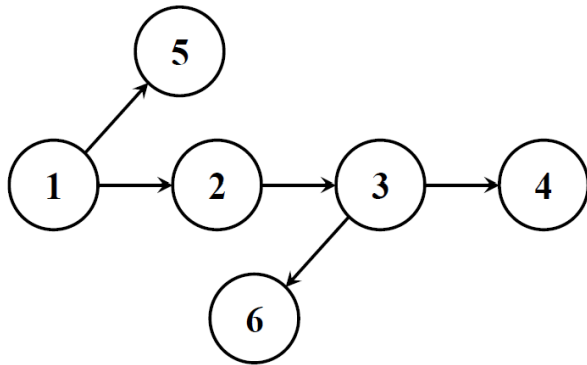


Fig 1: Example of connection between brain neurons.

Prior to PDC analysis, the EEG signals were preprocessed to remove artifacts based on methods proposed by (16). The PDC functional connectivity can be calculated from an interval of 1 second to ensure the local stationarity of the EEG signal. However, since the calculation duration is 1 minute, for each trial, the PDC results using the threshold of 0.2 were calculated for each second and averaged to determine the active channels during the task (Fig. 2).

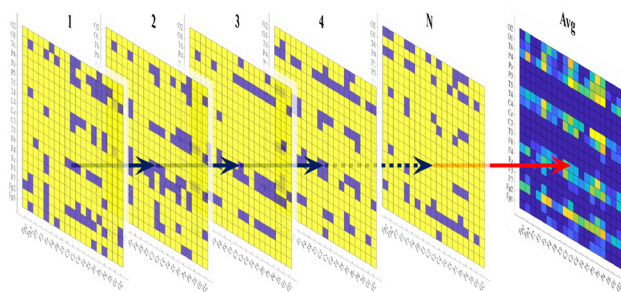


Fig 2: Average PDC functional connectivity for selected time N seconds (16).

Two software programs were used to perform the PDC analysis, i.e., MATLAB and Python. In this paper, the results from MATLAB are discussed, as both languages provide similar outcomes, albeit with a slight difference in methodologies. In Python, a third-party library's algorithm was utilized.

The PDC output was visualized as heatmaps (19x19 matrices) and colormaps of active brain regions. In these heatmaps, the color scale represents the strength of connectivity between electrode pairs, with warmer colors (e.g., red or yellow) indicating stronger directed connectivity values closer to the threshold of 1, and cooler colors (e.g., blue) representing weaker or absent connections near 0. When activation occurs, we

expect to see clusters of warmer colors corresponding to electrodes located in brain regions actively engaged during the task, while baseline or less involved regions appear with cooler shades. This visualization helps identify which regions act as strong information senders or receivers during mental arithmetic.

RESULTS

Fig. 3 illustrates EEG signals captured during a mental arithmetic task in one of the subjects, alongside the corresponding active brain regions. At selected time points, the figure highlights prominent activation in the frontal region, indicating this area's consistent engagement during the task.

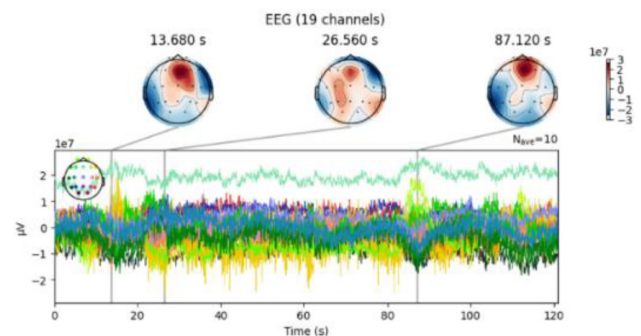


Fig 3: Example of EEG signals and active brain region during mental arithmetic task.

Fig. 4 presents averaged PDC heatmaps (19x19) and EEG active destination colormaps from the same subject during trial 1. These heatmaps and colormaps demonstrate increased activation in specific electrodes, particularly Fp2, F7, F8, T3, and T4, pointing to active engagement in both frontal and temporal lobes during the arithmetic activity. Similar activation patterns were observed across other trials and subjects. Notably, while frontal and temporal regions showed the most consistent activation, the occipital lobe was also occasionally active in certain trials or individuals. This suggests a recurring but varied brain region engagement pattern depending on the subject and potentially their problem-solving strategies

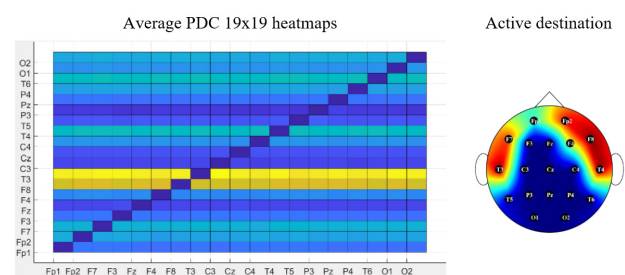


Fig 4: Average PDC functional connectivity and active destination.

DISCUSSION

The observed frontal lobe activation supports previous findings that associate this region with cognitive processes involved in mental arithmetic and mathematical cognition (17). The frontal cortex is responsible for decision-making, attention control, and working memory—all essential during complex mental tasks such as arithmetic calculations.

Furthermore, the involvement of the temporal lobe, specifically areas like T3 and T4, aligns with its established role in memory retrieval and language processing (18). Arithmetic operations often require the recall of numeric facts and procedural steps, making this region's activation relevant and expected.

Interestingly, occasional occipital lobe activation, though not directly tied to numerical computation, may reflect the use of visual strategies by some subjects. This is consistent with literature suggesting that when individuals visualize numbers, shapes, or equations, they may engage visual processing areas of the brain (18-20). This visual imagery can enhance the ability to mentally manipulate numerical information, making it a useful cognitive aid during arithmetic tasks.

The variability in occipital lobe activation across subjects underscores the individual differences in cognitive strategy. Some subjects may rely more on symbolic reasoning, while others may benefit from mental visualization. This highlights the importance of considering personalized cognitive approaches in future brain-computer interface designs or neurofeedback training program

CONCLUSION

This paper utilized functional connectivity during mental arithmetic tasks using the multivariate autoregressive model and short-time PDC analysis. By employing a third-party library's algorithm in Python and verifying results with MATLAB simulations, the study ensured the accuracy of its findings. The analysis revealed that brain connectivity active destination regions during arithmetic tasks using EEG-based PDC are similar to those found using fMRI. However, the posterior region is missing in the network of regions among the subjects involved when performing average PDC even-though the activation was seen in a few 1 second PDC results. The absence of parietal lobe activation during subtraction or other mental arithmetic tasks is unusual because the parietal lobe, particularly the intraparietal sulcus, is typically heavily involved in numerical processing and arithmetic. The absence of parietal activation could result from limitations in the average PDC technique employed in the experimental design. Poor sensitivity in detecting subtle neural signals might cause underreporting of parietal involvement. Another reason would be the

posterior region is involve in early processing thus may appear in source activation of PDC functional connectivity rather than in active destination, which is not covered in the paper. However, in some cases, particularly when parietal lobe activation is missing, the frontal lobes may compensate by taking on a larger role in the task, particularly in working memory, executive control, and decision-making processes. This could happen in individuals with more developed executive functions or when they use alternate problem-solving strategies that rely less on numerical manipulation. Further study is needed in this aspect.

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